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PROPAGATION OF WEAK PERTURBATIONS IN CRACKED POROUS MEDIA†

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A three-velocity, three-pressure mathematical model is proposed which enables one to study wave processes in the case of a double porosity, deformable, fluid-saturated medium. This model takes account of the differences in the velocities and pressures in pore systems of different characteristic scales of the pores, fluid exchange between these pore systems and the unsteady forces due to interphase interactions. It is established that a single transverse and three longitudinal waves: one deformation wave and two filtration waves, propagate in such a medium. The existence of two filtration waves is associated with the two different characteristic scales of the pores and the difference in the velocities and pressures of the fluid in these pore systems. The filtration waves decay considerably more rapidly than the deformation and transverse waves. The velocities of the deformation and transverse waves are mainly determined by the elastic moduli of the skeleton. The velocity and decay of the first filtration wave depend strongly on the intensity of the interphase interaction force while the velocity of the second filtration wave depends strongly on the rate of mass exchange between the pores and the cracks. The rate of decay of the second filtration wave is significantly higher than that of the first filtration wave. © 2000 Elsevier Science Ltd. All rights reserved.

The majority of papers dealing with cracked porous media are concerned with investigating seepage processes [1–15], but the special features of the propagation of waves in such media remain insufficiently studied [16–20].

1. A MEDIUM WITH DOUBLE POROSITY. REDUCED STRESS. THE MASS AND MOMENTUM BALANCE EQUATIONS

To study the process of the propagation of linear waves, we shall construct a model of a medium with double porosity which takes account of the differences in the velocities and pressures in the fluid phase contained in the pores and in the cracks as well as the interchange of fluid between them. We shall distinguish between the solid phase (indicated by the subscript s), the fluid in the pores (p) and the fluid in the cracks (f). Quantities referring to all of the fluid will be labelled with the subscript l. We shall assume that the mean radius of the primary pores a_p, the mean half-width of the cracks a_f and half the average size of the porous block a_b are the linear scales of the medium.

In the case of a conventional porous medium, the reduced (or effective) stress σ_s is defined by the equality

$$\sigma_{s*} = \alpha_s(\sigma_s - \sigma_l); \quad \sigma = \sigma_{s*} + \sigma_l \tag{1.1}$$

Here, α_s is the volume of the solid phase, σ_s and σ_l are the averaged true stresses within the solid and the liquid phases, respectively, and σ is the total stress in the medium. However, if account is taken of the fact that, in a medium with double porosity, the fluid pressures in the primary and secondary pores can be different, it is possible to express the mean stress σ_l in the fluid in terms of the stresses in the primary pores σ_p and the secondary pores σ_f as follows:

$$\sigma_l = \frac{\alpha_p\sigma_p + \alpha_f\sigma_f}{\alpha_p + \alpha_f} \tag{1.2}$$

We now write out the equations for the conservation of mass and momentum [21, 22] for the solid phase, for the fluid in the pores and in the cracks, taking account of relations (1.1) and (1.2)

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$$\frac{\partial\rho_s}{\partial t}+\nabla^k\rho_s v_s^k=0,\quad \frac{\partial\rho_p}{\partial t}+\nabla^k\rho_p v_p^k=-q,\quad \frac{\partial\rho_f}{\partial t}+\nabla^k\rho_f v_f^k=q$$

$$\rho_s\frac{dv_s^k}{dt}=-\alpha_s\nabla^k p_l+\nabla^j\sigma_{s*}^{jk}+F_p^k+F_f^k,\quad p_l=\frac{\alpha_p p_p+\alpha_f p_f}{\alpha_p+\alpha_f} \tag{1.3}$$

$$\rho_p\frac{dv_p^k}{dt}=-\alpha_p\nabla^k p_p-F_p^k-qv_p^k,\quad \rho_f\frac{dv_f^k}{dt}=-\alpha_f\nabla^k p_f-F_f^k+qv_f^k$$

Here, α_j, ρ_j and v_j are, respectively, the volume, the reduced density and the velocity of the jth phase, p_p and p_f are the pressures in the pores and in the cracks, q is the rate of exchange of fluid between the pores and the cracks, F_p and F_f are the forces of interaction between the solid phase and the fluid in the pores and the solid phase and the fluid in the cracks respectively, and the superscripts correspond to the coordinates.

It is assumed that the averaged viscous stress tensor in the fluid is negligibly small and that the fluid viscosity only manifests itself in the processes of the interaction of the fluid and the skeleton and fluid exchange between the pore systems.

In the case of the true densities ρ_j° and the phase volumes α_j, we have

$$\rho_j=\alpha_j\rho_j^\circ,\quad j=s,p,f;\quad \alpha_s+\alpha_p+\alpha_f=1 \tag{1.4}$$

In the case of steady-state harmonic oscillations with frequency ω, we adopt the expressions for the interphase forces F_p and F_f in (1.3) in the form [21, 22]

$$F_j=F_{mj}+F_{\mu j},\quad j=p,f \tag{1.5}$$

$$F_{mj}^k=\frac{1}{2}\eta_{mj}\alpha_s\alpha_j\rho_l^\circ i\omega(v_j^k-v_s^k),\quad F_{\mu j}^k=\alpha_s\alpha_j\mu_l f_j(\omega)(v_j^k-v_s^k)$$

$$f_j(\omega)=\eta_{\mu j}a_j^{-2}+\eta_{Bj}a_j^{-1}\sqrt{2\rho_l^\circ\omega/\mu_l}\,(1+i)$$

Here, F_{mj} is the force due to the inertial interaction of the phases [22], $F_{\mu j}$ is the total force of viscous friction, taking account of an analogue of the Basset force which arises due to transient effects, i is the square root of -1, μ_l is the dynamic viscosity of the fluid, a_j is the characteristic pore size and η_{mj}, $\eta_{\mu j}$, η_{Bj} are coefficients which depend on the structure of the porous medium.

A cross flow of fluid from one pore system into the other, that is, an exchange of fluid between the pore systems, is possible when the pressures p_p and p_f in the primary and secondary pores are different. The expression

$$q=\eta_q\frac{\rho_l^\circ k_p}{\mu_l}\frac{p_p-p_f}{a_b^2} \tag{1.6}$$

obtained under the assumption that the fluid flow is inertialess [4], is usually used for the rate of fluid exchange q when studying the seepage of a fluid in media with double porosity. Here, k_p is the permeability of the primary pore system and η_q is a dimensionless constant which characterizes the geometry of the medium. Note that the permeability k_p is expressed in terms of the coefficient for the steady force of viscous drag $F_{\mu p}$ namely $k_p=\alpha_p a_p^2/(\alpha_s\eta_{\mu p})$. If, however, allowance is also to be made for transient effects in the fluid flow, then, instead of $a_p^2/\eta_{\mu p}$, it is necessary to take the coefficient for the case of the total force of viscous drag $1/f_p(\omega)$. Hence, we write the expression for the rate of fluid exchange between the pore systems, q, in the form

$$q=\eta_q\frac{\alpha_p}{\alpha_s}\frac{\rho_l^\circ}{\mu_l f_p(\omega)}\frac{p_p-p_f}{a_b^2} \tag{1.7}$$

2. THE EQUATIONS OF STATE FOR THE ELASTIC SKELETON OF THE MEDIUM. THE EQUATIONS OF STATE OF THE PHASES

In deriving the equation which determines the behaviour of the skeleton of the porous medium, we shall adopt an approach which was previously described in [2, 21], according to which the tensor for the macrodeformations of the solid phase ε_s is defined as the sum of the averaged tensor of the microdeformations of the skeletal material ε_s° and the tensor of the effective deformations ε_{s*} due to the displacements of the grains with respect to one another

$$\varepsilon_s = \varepsilon_s^\circ + \varepsilon_{s^*}; \quad \frac{d}{dt}\varepsilon_s^{kl} = \frac{1}{2}(\nabla^k v_s^l + \nabla^l v_s^k) \tag{2.1}$$

It is natural to assume that, in a medium with double porosity, the effective deformation ε_{s*} occurs both as a result of the relative displacements of the granules within a porous block and as a result of the displacement of the blocks themselves with respect to one another. Suppose ε_{s*p} is the effective deformation due to the relative displacements of the granules within blocks and ε_{s*f} is the effective deformation due to the displacement of the blocks. We shall assume that

$$\varepsilon_{s^*} = \varepsilon_{s^*p} + \varepsilon_{s^*f}, \quad \varepsilon_{s^*p}^{kl} = (1-\eta)\varepsilon_{s^*}^{kl}, \quad \varepsilon_{s^*f}^{kl} = \eta\varepsilon_{s^*}^{kl} \tag{2.2}$$

where η is a certain (as yet unknown) scalar quantity.

We next assume that the reduced (effective) stress σ_{s*} in the mixture depends linearly on the effective deformation ε_{s*}. Then, taking account of equality (2.2), we have

$$\sigma_{s^*} = \sigma_{s^*p} + \sigma_{s^*f} \tag{2.3}$$

where σ_{s*p} is the effective intergranular stress, which depends linearly on ε_{s*p} and σ_{s*f} is the effective stress between the blocks which depends linearly on ε_{s*f}. We assume that each of the stresses σ_{s*p}, σ_{s*f} is proportional to the difference between the averaged true stresses in the phases with a certain coefficient of proportionality

$$\sigma_{s^*j} = \varkappa_j(\sigma_s - \sigma_j), \quad j = p, f \tag{2.4}$$

Substituting the last expressions (2.4) into (2.3) and taking account of relations (1.1) and (1.2), we find the values of the coefficients $\varkappa_p$ and $\varkappa_f$. As a result, we obtain

$$\sigma_{s^*j} = \frac{\alpha_s\alpha_j}{\alpha_p + \alpha_f}(\sigma_s - \sigma_j), \quad j = p, f \tag{2.5}$$

The effective expressions for σ_{s*p} and σ_{s*f} can be interpreted as follows: σ_{s*p} is the part of the effective stress σ_{s*} which is responsible for the transfer of momentum through the solid phase across the contacts between the granules of a porous block and σ_{s*f} is the part of σ_{s*} which is responsible for the transfer of momentum across the contacts between the blocks. In the limiting cases when $\alpha_p = 0$ (a cracked medium) or $\alpha_f = 0$ (a conventional porous medium), we have $\sigma_{s*p} = 0$, $\sigma_{s*} = \sigma_{s*f}$ or $\sigma_{s*f} = 0$, $\sigma_{s*} = \sigma_{s*p}$, respectively.

For small deformations of the medium, we assume that each of the relations σ_{s*j}^{kl} (ε_{e*j}^{mn}) is described by Hooke's law with certain elastic moduli which characterize the skeleton of the porous medium (δ^{kl} is the Kronecker delta)

$$\sigma_{s^*j}^{kl} = \alpha_s(\tilde{\lambda}_{*j}\varepsilon_{s^*j}^{mm}\delta^{kl} + 2\tilde{\mu}_{*j}\varepsilon_{s^*j}^{kl}), \quad j = p, f \tag{2.6}$$

Hooke's law holds for the deformations of the material of the grains (λ_s and μ_s are the elasticity moduli of the solid phase material)

$$\varepsilon_s^{\circ kl} = \frac{1}{2\mu_s}\left(\sigma_s^{kl} - \frac{\lambda_s}{3\lambda_s + 2\mu_s}\sigma_s^{mm}\delta^{kl}\right) \tag{2.7}$$

Taking account of relations (1.2), (2.2), (2.5) and (2.7) and the equalities $\delta_{kl}^p = -\delta^{kl}p_p$, $\sigma_f^{kl} = -\delta^{kl}p_f$, we express ε_{s*j}^{mn} in terms of σ_{s*j}^{kl} $(j = p, f)$. Substituting these expressions into (2.6) and solving them for σ_{s*j}^{kl}, σ_{s*j}^{kl}, we obtain

$$\begin{aligned} \sigma_{s^*p}^{kl} &= (1-\eta)\alpha_s[\lambda_{*p}\varepsilon_s^{mm}\delta^{kl} + 2\mu_{*p}\varepsilon_s^{kl} + \nu_{*p}p_p\delta^{kl}] \\ \sigma_{s^*f}^{kl} &= \eta\alpha_s[\lambda_{*f}\varepsilon_s^{mm}\delta^{kl} + 2\mu_{*f}\varepsilon_s^{kl} + \nu_{*f}p_f\delta^{kl}] \end{aligned} \tag{2.8}$$

where

$$\lambda_{*j}=\tilde{\lambda}_{*j}\left[1+O\left(\frac{\tilde{\lambda}_{*j}}{\lambda_s},\frac{\tilde{\mu}_{*j}}{\mu_s}\right)\right],\quad \mu_{*j}=\tilde{\mu}_{*j}\left[1+\frac{\tilde{\mu}_{*j}}{\mu_s}\right]^{-1}$$
$$\nu_{*j}=\frac{\lambda_{*j}+2\mu_{*j}/3}{\lambda_s+2\mu_s/3},\quad j=p,f \tag{2.9}$$

For $\sigma_{s*}=\sigma_{s*p}+\sigma_{s*f}$, we can write

$$\sigma_{s*}^{kl}=\sigma_s[\lambda_*\varepsilon_s^{mm}\delta^{kl}+2\mu_*\varepsilon_s^{kl}+(1-\eta)\nu_{*p}p_p\delta^{kl}+\eta\nu_{*f}p_f\delta^{kl}] \tag{2.10}$$
$$\lambda_*=(1-\eta)\lambda_{*p}+\eta\lambda_{*f},\quad \mu_*=(1-\eta)\mu_{*p}+\eta\mu_{*f}$$

Note that, if an equation of state of the skeleton is derived which does not take account of the more complex structure of a medium with double porosity and the difference in the pressures p_p and p_f (that is, as this equation is derived in the case of a conventional porous medium [2, 21]), a formula is obtained after analogous arguments which only takes account of certain "mean" moduli of elasticity of the skeleton λ_* and μ_* and the mean pressure in the fluid p_l

$$\sigma_{s*}^{kl}=\alpha_s[\lambda_*\varepsilon_s^{mm}\delta^{kl}+2\mu_*\varepsilon_s^{kl}+\nu_*p_l\delta^{kl}],\quad \nu_*=\frac{\lambda_*+2\mu_*/3}{\lambda_s+2\mu_s/3} \tag{2.11}$$

This formula can also be considered as an equation of state for the skeleton of the medium but, in the case of a medium with double porosity, it is less detailed than (2.10). If one takes λ_* and μ_*, as defined above, and equates the coefficients of p_p and p_f in (2.10) and (2.11), it is found to be possible to find the as yet unknown quantity η

$$\eta=\left(1+\frac{\alpha_p}{\alpha_f}\frac{\nu_{*f}}{\nu_{*p}}\right)^{-1} \tag{2.12}$$

Hence, if the behaviour of the skeleton of a medium with double porosity is assumed to be elastic, it is necessary to specify the four elasticity moduli λ_{*p}, μ_{*p}, λ_{*p} and μ_{*p}. The equation of state (2.10) relates the reduced stress σ_{*s}, the macrodeformations of the skeleton ε_s and the pressures in the primary and secondary pores p_p and p_f; the quantities ν_{*p}, ν_{*f} and η are uniquely defined by the coefficients λ_{*p}, μ_{*p}, λ_{*p} and μ_{*p} and the volumes α_p, α_f in accordance with relations (2.9) and (2.12). Note that, in the limiting cases when $\alpha_p=0$ or $\alpha_f=0$, we have $\eta=1$ or $\eta=0$ in (2.12), respectively, and the equation of state (2.1) is identical with the equation of state for a conventional elastic porous medium with elasticity moduli λ_{*f}, μ_{*f} or λ_{*p}, μ_{*p}.

When a more exact quantitative description of the decay of the waves is required, a more complex equation of state can be constructed for the solid state by taking account, for example, of the viscoelastic behaviour of the skeleton. In this case, the dissipation of the kinetic energy due to the friction between the grains of the skeleton during its deformation will be taken into account.

Note that, in [16, 17], an equation of state of a medium was used which is described in terms of the pressures in the pores and cracks and the total stress in the medium, that is, without making use of the concept of an effective stress. In [19, 20], the equations of state and the motions of a cracked porous medium were obtained from the corresponding equations which describe the process on a microscale by the spatial averaging method. The final equations are written in terms of displacements, which means that they are cumbersome and difficult to understand.

The equations of state for the material of the solid phase and for the fluid in the pores and in the cracks can be taken in the form

$$p_s'=K_s\overset{\circ}{\rho}{}_s'/\overset{\circ}{\rho}_{s0}(K_s=\lambda_s+2\mu_s/3),\quad p_j'=K_l\overset{\circ}{\rho}{}_j'/\overset{\circ}{\rho}_{l0},\quad j=p,f \tag{2.13}$$

where K_s and K_l are the elasticity moduli for the all-round bulk compression of the solid phase material and the fluid (henceforth, a zero subscript denotes the unperturbed value of a quantity, $u'=u-u_0$).

3. THE CONDITIONS FOR COMPATIBLE DEFORMATION OF THE PHASES

The relation between the pressures p_{s*}, p_s, p_l, which follows from (1.1)

$$p_{s*} = \alpha_s(p_s - p_l), \quad p_{s*} = -\sigma_{s*}^{mm}/3 \tag{3.1}$$

or the equation for the change in porosity, which is a consequence of (3.1) and, in the case of a medium with an elastic skeleton, has the form [2, 21]

$$\frac{\alpha_s'}{\alpha_s} = (1-\nu_*)\left(\frac{\rho_s'}{\rho_s} - \frac{p_l'}{K_s}\right) \tag{3.2}$$

is used for the closure of the system of equations of motion of a conventional porous medium.

In the case of a medium with double porosity, not just one but two conditions for the compatible deformations of the phases are required for the closure of the system of equations (this is associated with the existence of two pressures in the fluid). The relation between the pressures p_{s*}, p_s, p_p, p_f, which follows from (1.1)

$$p_{s*} = \alpha_s(p_s - p_l), \quad p_{s*} = -\sigma_{s*}^{mm}/3 \tag{3.3}$$

can be taken as the first of these conditions.

Equation (3.2) is used to obtain the second condition of compatible deformation. We will derive an equation for the change within a porous block. For this purpose, in (3.2), it is necessary to replace p'_l by p'_p, ν_* by ν_{*p}, α_s by $\alpha_s/(\alpha_s + \sigma_p)$ and $\rho_s = \alpha_s\rho_s^\circ$ by $\rho_s/(\alpha_s + \alpha_p) = \alpha_s\rho_s^\circ/(\alpha_s + \alpha_p)$ since these quantities now refer not to unit volume of the medium but to the volume of a porous block in unit volume of the medium. The equation

$$\frac{\alpha_s'}{\alpha_s} + \frac{\alpha_f'}{1-\alpha_f}\nu_{*p} = (1-\nu_{*p})\left(\frac{\rho_s'}{\rho_s} - \frac{p_p'}{K_s}\right) \tag{3.4}$$

is obtained as a result.

The system of equations (1.3), (1.4), (2.1), (2.3), (2.8), (1.2), (2.13), (3.3) and (3.4), which has been obtained with the given expressions for mass and momentum exchange (1.5) and (1.7) and the specified elasticity moduli of the skeleton λ_{*p}, μ_{*p}, λ_{*f}, μ_{*f} is closed and can be used for the linear analysis of the wave processes in a medium with double porosity.

4. THE PROPAGATION OF LINEAR WAVES IN A CRACKED POROUS MEDIUM

We will now consider the motion of a medium corresponding to the propagation of one-dimensional monochromatic waves, that is, when

$$v_j^x = v_j = V_j \exp(i\omega t - ikx), \quad v_j^y = v_j^z = 0$$

in the case of longitudinal waves, and

$$v_j^z = v_j = V_j \exp(i\omega t - ikx), \quad v_j^x = v_j^y = 0$$

in the case of transverse waves ($j = s, p, f$; t is the time and x, y and z are Cartesian coordinates), ω is the angular velocity and k is a complex wavenumber. The dispersion relations for the longitudinal and transverse waves are obtained by substituting the solutions in the above-mentioned form into the linearized system of equations of motion of the medium and equating the determinant of this system to zero.

It was found that the dispersion relation for the longitudinal waves is an algebraic equation of the third degree in k^2 and, in the case of the transverse waves, an equation of the first degree in k^2. It follows from this that a single transverse wave and three types of longitudinal waves can propagate in the medium. The velocity $C^{(m)}$ and the decay $\delta^{(m)}$ of each of these waves are determined after solving the dispersion relation $k^{(m)} = k^{(m)}(\omega)$ using the formulae

$$C^{(m)} = \omega/\mathrm{Re}\, k^{(m)}, \quad \delta^{(m)} = -\mathrm{Im}\, k^{(m)} \tag{4.1}$$

where $m = 1, 2, 3$ corresponds to the longitudinal waves and $m = 4$ corresponds to the transverse wave.

The conclusion has also been previously drawn [16, 17] that there are three types of longitudinal waves and a single transverse wave in a cracked porous medium. One of the longitudinal waves is associated with the elastic properties of the skeleton, the second with the pore space and the third with the space of the cracks. The processes involved in the propagation of monochromatic waves in a cracked porous medium saturated with two different fluids have been studied in [19, 20] and it has been shown that there are four different types of longitudinal waves. Here, the first and third waves are analogous to the fast and slow waves in a porous medium, the second wave arises due to the existence of cracks, and the fourth wave is associated with the pressure difference in the fluids in the porous blocks.

Here, it is necessary to recall that two types of longitudinal waves exist in a conventional porous medium which are due to the two different mechanisms for momentum transfer: through the fluid and directly through the solid phase. The natural question therefore arises as to how the appearance of a third longitudinal wave in a medium with double porosity is to be explained. In order to elucidate the mechanism by which the additional wave occurs, detailed consideration was given to the propagation of waves in a cracked porous medium with an incompressible and undeformable skeleton, that is, when $v_s = 0$ and $\alpha_j = \mathrm{const}$.

In this case, the system of equations of motion, neglecting the inertial terms in the momentum balance equations, is equivalent to the equations in [1], which are used to describe seepage processes in a cracked porous medium. It has been found that, in such a medium, two longitudinal waves propagate through the fluid (that is, there is also an additional longitudinal wave in this case) and that the velocity and decay of these waves are determined by the interphase forces F_p and F_f and the rate of fluid exchange between the pore systems, q. If interphase forces are neglected in the equations of motion but mass exchange between the pore systems is taken into account, ($F_p = F_f = 0$, $q \neq 0$), it then follows from the dispersion relation that, in this case, one of the waves propagates at the speed of sound C_l in the pure fluid without attenuation. The other wave propagates at a lower velocity and decays as it propagates. If, in the initial equations, no account is taken of the cross flow of the fluid between the pore systems but the interphase forces (F_p, $F_f \neq 0$, $q = 0$) are taken into account, the velocity of each of the waves is less than C_l and both waves decay during propagation. If, however, both fluid exchange and interphase interactions are neglected ($F_p = F_f = 0$, $q = 0$), it is found that, in this case, each of the two waves propagates at the speed of sound in the pure fluid C_l without attenuation. It is clear that an identical type of propagation of these two waves has been obtained due to the fact that no account whatsoever has been taken of the fluid viscosity in the latter case and the effect of this depends very much on the characteristic scales of the inhomogeneity of the medium. The mean velocity of motion of a viscous fluid and the pressure depends on the size of the pores. It is therefore necessary when treating a medium with two characteristic pore sizes to take account of the difference in the velocities and pressures in each of the pore systems as a consequence of the effect of the fluid viscosity.

5. RESULTS OF THE NUMERICAL INVESTIGATION

Using the dispersion relations constructed, the phase velocity and the linear damping decrement were calculated for the waves of each type in accordance with formulae (4.1). The calculation was carried out for a medium in which the solid phase material was quartz and the fluid was water. The initial pressure in the medium $p_0 = 0.1$ MPa and the parameters for the phases were as follows: $\rho_{s0}^\circ = 2500$ kg/m^3, $K_s = 5 \times 10^{10}$ Pa, $\rho_{l0}^\circ = 1000$ kg/m^3, $K_l = 2.25 \times 10^9$ Pa and $\mu_l = 10^{-3}$ Pa s. The principal parameters of the cracked porous medium were: $\alpha_p = 0.25$, $\alpha_f = 0.01$, $a_p = 0.01$ mm, $a_f = 0.1$ mm and $a_b = 2$ mm. It was checked that the continuity condition (the wavelength must be greater than the characteristic dimension of the inhomogeneity of the medium) was satisfied during the calculations. Over the frequency range being considered, this condition holds in the case of type 1 and type 2 longitudinal waves and the type 4 transverse wave and it is also satisfied in the case of a type 3 longitudinal wave for frequencies up to 10^5 s^{-1}.

The phase velocity and the linear damping decrements of the waves which propagate in a cracked porous medium with an incompressible and undeformable skeleton are shown in Fig. 1. It has been pointed out above that two types of filtration waves can propagate in such a medium. The velocities of both waves are small at low frequencies. The wave with the greater velocity (shown by the solid line) is characterized by a significantly smaller decay compared with the second slower wave (the dashed line).

The velocities and decays of the longitudinal wave (type 1–3) and transverse wave (type 4) in a cracked porous medium, calculated taking account of the compressibility and deformability of the skeleton, are shown by the solid lines in Fig. 2. The velocities of the two slower waves at low frequencies are close to zero, that is, these are two

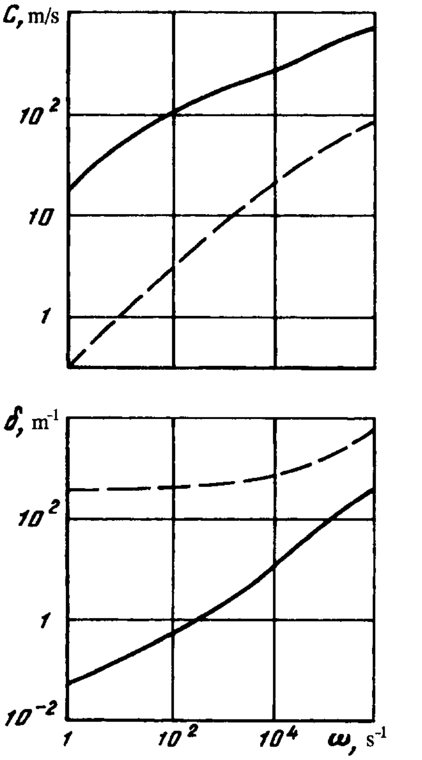

Fig. 1.

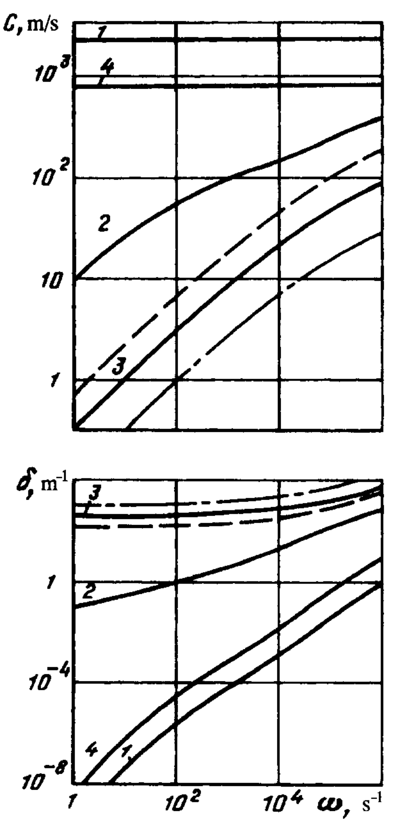

Fig. 2.

filtration waves. The velocities of the deformation wave (type 1) and the transverse waves (type 4) are practically independent of the frequency and are mainly determined by the elasticity moduli of the skeleton of the medium. The deformation and transverse waves (types 1 and 4) decay far more weakly than the filtration waves (types 2 and 3). If one compares Figs 1 and 2, it becomes obvious that waves of types 2 and 3, which are depicted by the solid lines in Fig. 2, correspond to the waves depicted by the solid and dashed line in Fig. 1. The deformation and transverse waves apparently do not exist in a medium with a rigid skeleton. The small values of the velocity of the type 2 wave in Fig. 2 compared with the wave corresponding to it in Fig. 1 (the solid line) are explained by taking account of the compressibility and deformability of the skeleton of the porous medium (this effect is analogous to the effect of the compressibility of the material of the tube walls when sound propagates in narrow tubes, which has been previously treated in detail [23]).

The effect of the rate of fluid exchange between the pore systems on the characteristics of the linear waves in a fractured porous medium (the dashed, solid and dot–dash lines correspond to η_q = 0.1; 0.5; 5) is also illustrated in Fig. 2. It is seen that a change in the rate of mass exchange only has an effect on the filtration wave of type 3. As η_q increases, the velocity of this wave decreases and the decay increases. A filtration wave of type 3 is therefore more conspicuous at a low rate of mass exchange, q. It disappears in the limit of an infinite rate of mass exchange q.

The effect of the other parameters of the model, the medium, and the interphase interaction forces on the characteristics of the propagation of linear waves in a medium with double porosity was also studied.

It was found that, in the transverse wave, the pressures in the fluid and in the primary and secondary pores remain constant, that is, $p_p^{(4)} = p_f^{(4)} = p_0$ (this follows from an analysis of the linearized system of equations) and only the shear components of the reduced stress tensor σ_{s*} change.

The nature of the change in the pressure perturbations

$$p_j^{(m)} = A_j^{(m)} \exp(i\omega t - ik^{(m)}x), \quad j = p, f$$

of the mean pressure of the fluid and the longitudinal reduced stress

$$p_j^{(m)} = \frac{\alpha_{p0} p_p + \alpha_{f0} p_f^{(m)}}{\alpha_{p0} + \alpha_{f0}}, \quad \sigma_{s*}^{(m)} = A_s^{(m)} \exp(i\omega t - ik^{(m)}x)$$

in each of the longitudinal waves was also investigated (here $A_p^{(m)}$, $A_f^{(m)}$, $A_s^{(m)}$ are the complex amplitudes, $m = 1, 2, 3$ and the primes on the pressures and stresses have been omitted). It was found that, in the frequency range ($1\ s^{-1} \leqslant \omega \leqslant 10^6\ s^{-1}$) being considered, the ratios of the amplitudes barely change and, in fact

$$\frac{p_l^{(1)}}{-\sigma_{s*}^{(1)}} \approx \text{const} > 0, \quad \frac{p_p^{(1)}}{p_f^{(1)}} \approx 1, \quad \frac{p_l^{(2)}}{-\sigma_{s*}^{(2)}} \approx -1, \quad \frac{p_p^{(2)}}{p_f^{(2)}} \approx 1, \quad \frac{p_l^{(3)}}{-\sigma_{s*}^{(3)}} \approx -1$$

and, in addition, Re $(p_p^{(3)}/p_f^{(3)}) < 0$.

The above-mentioned constant is determined by the parameters of the unperturbed state of the medium, that is, by the initial volumes, densities, etc. (in particular, the value $p_l^{(1)}/(-\sigma(1)_*) \approx 2.4$) was obtained for the fractured porous medium with the equilibrium parameters used here). In the case of a third wave, the ratio $p_p^{(3)}/p_f^{(3)}$ is variable when $1\ s^{-1} \leqslant \omega \leqslant 10^6\ s^{-1}$ and the inequality of the pressures $p_p^{(3)}$ and $p_f^{(3)}$ follows from this.

It follows from the results of the calculations that, in the case of the deformation wave (type 1), both the skeleton and the fluid simultaneously experience either compression or dilatation and the fluid pressures in the pore systems are equal. In the case of the propagation of filtration waves (of type 2 and type 3), the change in the total stress in the medium is approximately equal to zero

$$\sigma^{(2)} = \sigma_{s*}^{(2)} - p_l^{(2)} \approx 0, \quad \sigma^{(3)} = \sigma_{s*}^{(3)} - p_l^{(3)} \approx 0$$

In a type 2 wave, as in a type 1 wave, the fluid pressures are equal in the two pore systems. Conversely, in a type 3 wave, the pressures are not the same in the two pore systems, that is, the propagation of this wave is accompanied by a cross flow of fluid from one pore system into the other. The very pronounced decay of a type 3 wave shows that the equalization of the pressures occurs very rapidly and, consequently, the process of fluid exchange ceases. Hence, in a porous medium with two characteristic pore scales, the appearance of a longitudinal wave of type 3 is explained by the difference in the fluid pressure in the pore systems.

In conclusion, we note that the possibility of the appearance of a filtration wave in a saturated porous medium was predicted theoretically in the 1950s [24–27] and experimental confirmation using specially prepared laboratory samples was obtained only in the 1980s [28] and quite recently using a natural rock sample. The second filtration wave in a medium with double porosity has a high rate of decay and its experimental observation is therefore problematical.

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